A unique Fock quantization for scalar fields in cosmologies with signature change

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Ambiguities in QFT

- The quantization of a classical system is **NOT univocally** defined. Even in linear field theory, one finds **infinitely many** Fock quantizations.

- There exist ambiguities in the choice of:

  - the **field description**

  - the **Fock representation** of the CCR's

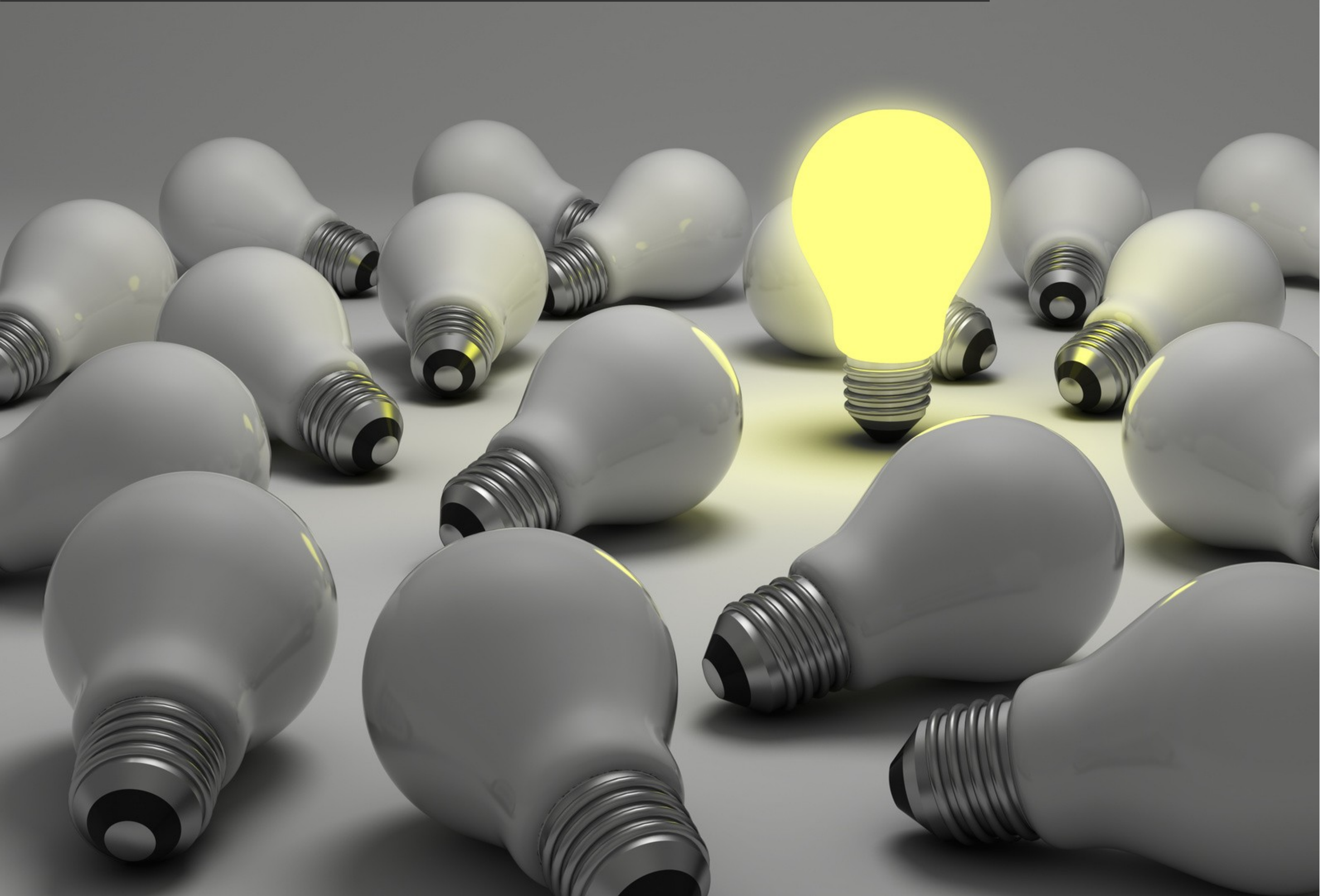
  which are not equivalent.

- In highly symmetric spacetimes, the invariance under the isometries of the background is enough to select a unique Fock quantization.

- For STATIONARY spacetimes, one can select a quantization with certain requirements on energy.

- In general, systems lack of sufficient symmetry. Recently, **UNIQUENESS** has been reached in some nonstationary scenarios by appealing to the unitarity of the dynamics, rather than to invariance.

Uniqueness criteria



Uniqueness criteria

Klein-Gordon field in ultrastatic spacetime, with **time-dependent** mass:

$$\varphi'' - \Delta\varphi + m^2(t)\varphi = 0$$

SPATIAL SYMMETRY INVARIANCE

+

UNITARY DYNAMICS

➡ select a **UNIQUE canonical pair** for the field.

➡ select also a **UNIQUE Fock representation** for the CCR's, for any (smooth) mass.

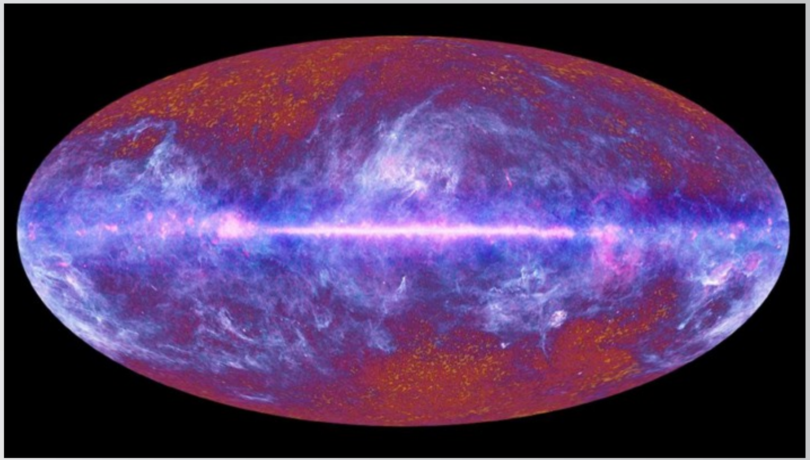
- The uniqueness result is valid for any spatial topology, and at least in any spatial dimension no larger than three.

Motivation: Fields with time dependent mass

$$\varphi'' - \Delta\varphi + m^2(t)\varphi = 0$$

RESCALED FIELDS in **FLAT COSMOLOGIES**
(conformal time)

COSMOLOGICAL PERTURBATIONS



- SCALAR PERTURBATIONS:
  Mukhanov-Sasaki variables (gauge invariant).
- PERTURBATIONS of a MASSIVE FIELD in a suitable gauge: asymptotic behavior.
- TENSORIAL PERTURBATIONS (gravitational waves).

Motivation: Generalized field equations

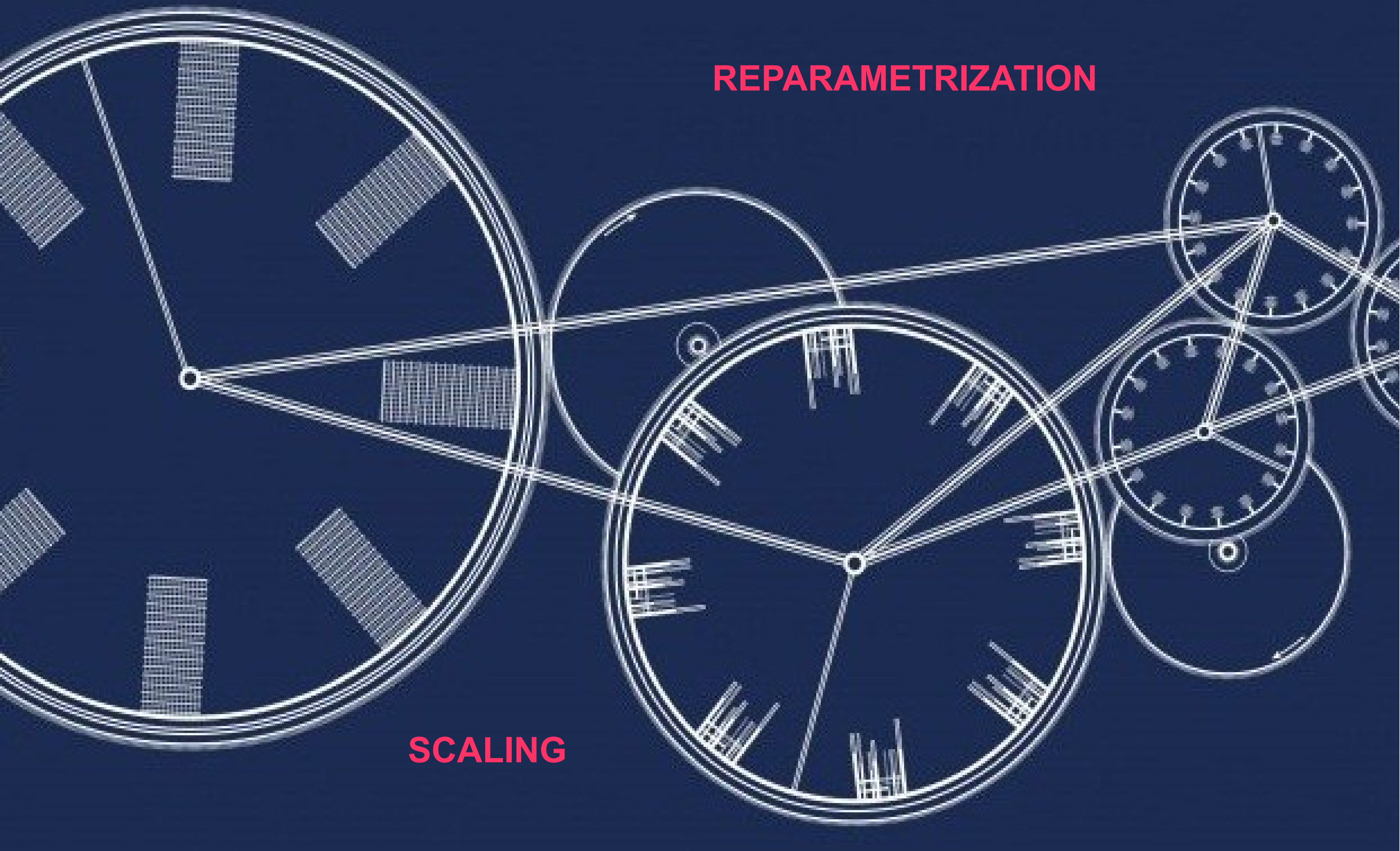
We want to generalize the class of field equations for which we can apply our UNIQUENESS results.

We would cover more general situations in cosmology, obtaining robust quantizations.

We will consider the most **general second-order differential equation** of KG type, preserving the spatial dependence only through the LB operator.

We would like to study situations with **“signature change”**. This kind of scenarios have received a lot of attention in Loop Quantum Cosmology recently.

Generalization of the field equations



Generalization of the field equations

$$\phi'' + c(t)\phi' - d(t)\Delta\phi + \tilde{m}^2(t)\phi = 0$$

$$\phi(t,\vec{x}) = f(t)\varphi(t,\vec{x})$$

SCALING

$$dT = g(t)\,dt, \qquad g(t) \neq 0$$

REPARAMETRIZATION

$$\varphi'' - \Delta\varphi + m^2(t)\varphi = 0$$

Up to time reversal, there is a **bijective correspondance**:

$$f(t) = C\,d(t)^{-1/4}\exp\left[-\frac{1}{2}\int^t c(\bar{t})\,d\bar{t}\right]$$

$$g(t) = s\sqrt{d(t)}, \qquad s = \pm$$

Generalization of the field equations

$$\phi'' + c(t)\phi' - d(t)\Delta\phi + \tilde{m}^2(t)\phi = 0$$

$$\phi(t,\vec{x}) = f(t)\varphi(t,\vec{x})$$

SCALING

$$dT = g(t)\,dt, \qquad g(t) \neq 0$$

REPARAMETRIZATION

$$\varphi'' - \Delta\varphi + m^2(t)\varphi = 0$$

The new **mass**:

$$m^2(t) = \frac{\tilde{m}^2(t)}{d(t)} - \frac{d''(t)}{4\mathrm{d}^2(t)} + \frac{5(d'(t))^2}{16\, d^3(t)} - \frac{c'(t)}{2\, d(t)} - \frac{c^2(t)}{4\, d(t)}$$

Generalization of the field equations

$$f(t) = C\,d(t)^{-1/4} \exp\left[-\frac{1}{2}\int^{t} c(\bar{t})\,d\bar{t}\right]$$

SCALING

$$g(t) = s\sqrt{d(t)}, \qquad s = \pm$$

REPARAMETRIZATION

$$m^2(t) = \frac{\tilde{m}^2(t)}{d(t)} - \frac{d''(t)}{4d^2(t)} + \frac{5(d'(t))^2}{16\,d^3(t)} - \frac{c'(t)}{2\,d(t)} - \frac{c^2(t)}{4\,d(t)}$$

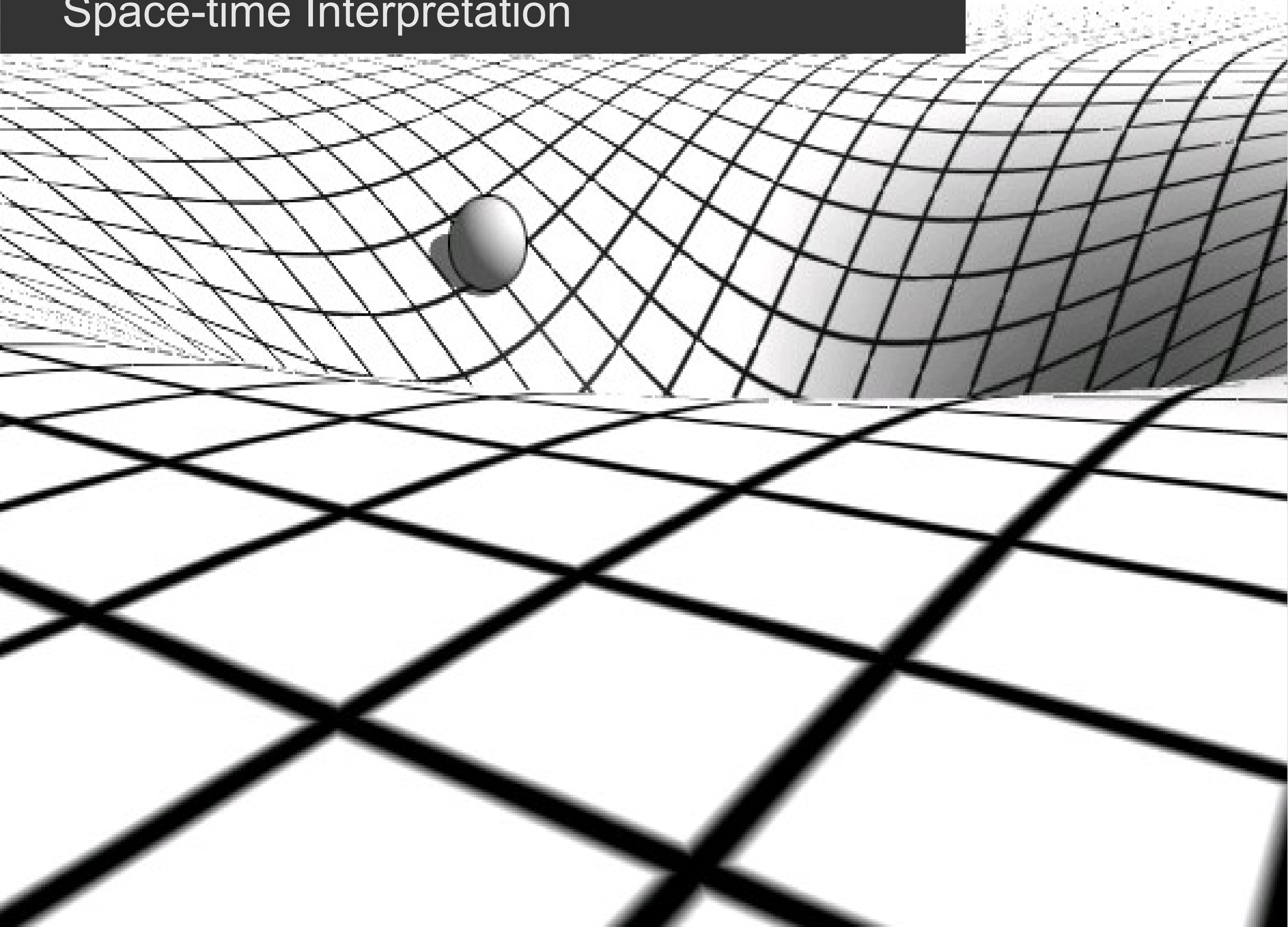
MASS

When the function $d(t)$ vanishes:

- The mass $m(t)$ explodes, in general.
- The scaling and the reparametrization are ill defined.

If it becomes negative, the new time parametrization turns imaginary.

Space-time Interpretation



Space-time Interpretation

Let us consider a **conformally ultrastatic spacetime**, with normal spatial sections:

$$ds^2 = -N^2(t)\,dt^2 + a^2(t)\,h_{ij}(x)\,dx^i\,dx^j$$

The considered field equations are the corresponding Klein-Gordon equations (of mass $\bar{m}(t)$) under the **univocal correspondence:**

$$a^4(t) = d(t)\exp\left[\int^t 2c(\bar{t})\,d\bar{t}\right]$$

$$N^4(t) = d^3(t)\exp\left[\int^t 2c(\bar{t})\,d\bar{t}\right]$$

$$\boxed{\phi'' + c(t)\phi' - d(t)\Delta\phi + \tilde{m}^2(t)\phi = 0}$$

Where: $\tilde{m}(t) = N(t)\,\bar{m}(t)$

Space-time Interpretation

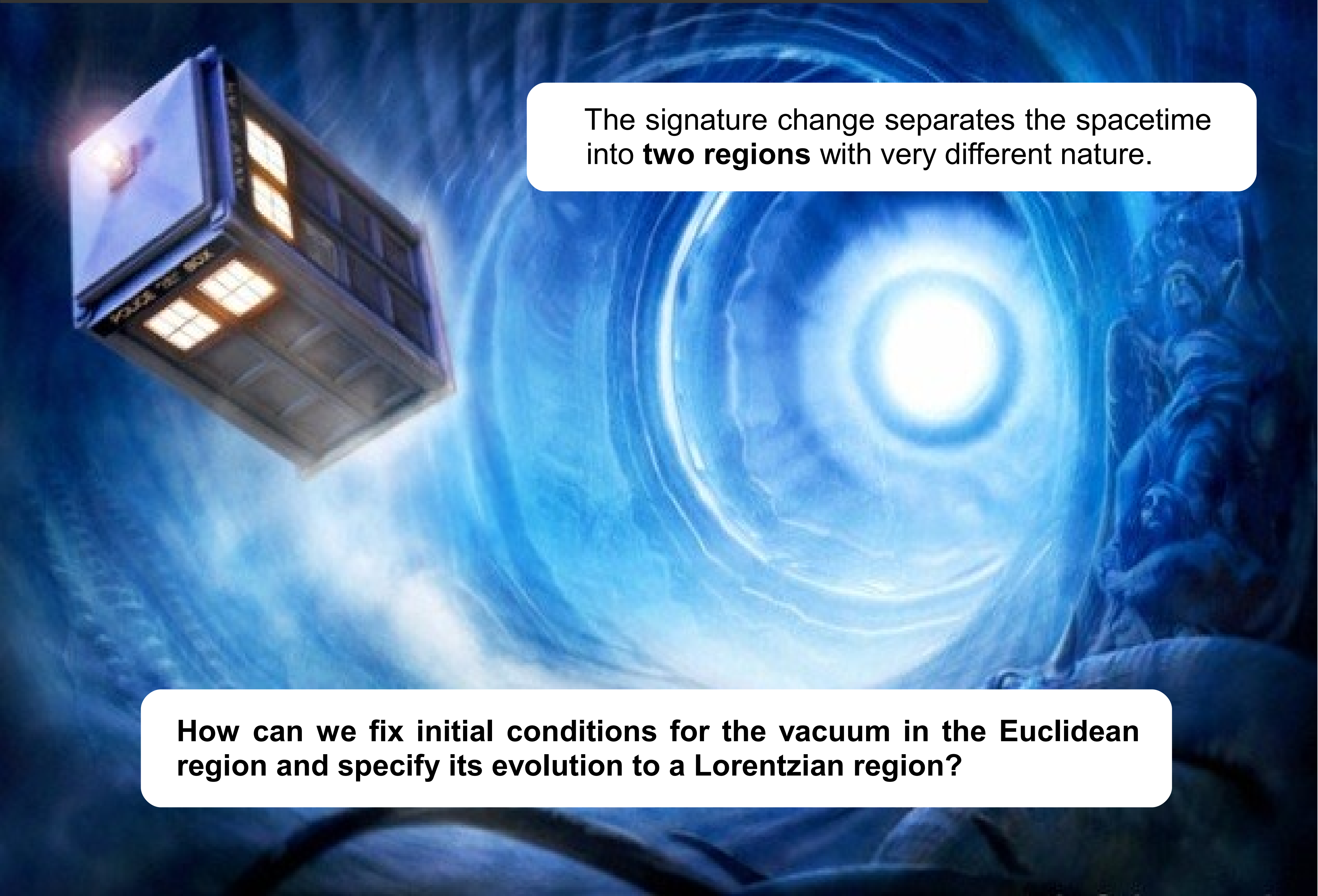
$$ds^2 = \left[-d(t)\, dt^2 + h_{ij}(x)\, dx^i\, dx^j\right] D\sqrt{|d(t)|}\, \exp\int_{t_d}^{t} c$$

- The metric **degenerates completely** when $d(t)$ vanishes.
- If we set $d(t_d)=0$, the metric becomes **Euclidean** in the region where $d(t)<0$.

$$ds^2: \qquad (-\ +\ +\ +) \longrightarrow (+\ +\ +\ +)$$

- From this perspective, it is more than a **signature change**. It involves a SINGULARITY where the scalar curvature explodes as $d^{-7/2}$.

Vacuum dynamics with signature change



The signature change separates the spacetime into **two regions** with very different nature.

How can we fix initial conditions for the vacuum in the Euclidean region and specify its evolution to a Lorentzian region?

Vacuum dynamics with signature change

We study the evolution of a fixed vacuum state in the **Euclidean** region:

i. We choose a complete set of solutions in the Lorentzian region $\left\{\varphi_n^{\pm}(T)\psi_n(\vec{x})\right\}$.

ii. Scaling by the invers of the scale factor and reparametrizating in terms of the time τ corresponding to the lapse $N^2=\epsilon a^6$, $\epsilon = \pm$, we find the set of **solutions** $\left\{\phi_n^{\pm}(\tau)\psi_n(\vec{x})\right\}$

$$\ddot{\phi} = -\epsilon\left[a^4\Delta\phi + a^6\bar{m}^2\phi\right]$$

iii. **Wick rotation** of the modes in the Euclidean regime $\phi_n^{\pm(E)} = \lim_{\tilde{\tau}\to i\tau}\phi_n^{\pm}(\tilde{\tau}).$

iv. The solutions can be expressed as a linear combination of these modes with coefficients $c_n^{\pm(E)}$ and $c_n^{\pm}$, respectively, for the Euclidean and Lorentzian regions.

v. We set the initial conditions at τ_0. We require **continuity** conditions of the field and its time derivative at the signature change instant, in which the **metric degenerates**.

Vacuum dynamics with signature change

- Imposing the continuity conditions, we obtain a linear system for each mode that relates the coefficients of the Euclidean and Lorentzian regions:

$$\begin{pmatrix} c_n^{+} \\ c_n^{-} \end{pmatrix} = \begin{pmatrix} -I_n^{(+-)} & -I_n^{(--)} \\ I_n^{(++)} & I_n^{(-+)} \end{pmatrix} \begin{pmatrix} c_n^{+(E)} \\ c_n^{-(E)} \end{pmatrix}$$

where $\quad I_n^{(r\,s)} = \lim_{\tau \to 0} \langle \phi_n^{r(E)}(\tau), \phi_n^{s}(\tau) \rangle, \qquad r, s = + \acute{o} -.$

Using that the modes are orthonormal under the KG-type product.

- The field φ with unitary evolution in the Lorentzian region:

$$\varphi = a(T) \sum_n \left(c_n^{+} \phi_n^{+}[\tau(T)] + c_n^{-} \phi_n^{-}[\tau(T)] \right) \psi_n(\vec{x}).$$

Vacuum dynamics with signature change

Starting only with *positive frequency* contributions in the Euclidean sector, $c_n^{+(E)}=0$, the corresponding combination in the Lorentzian region has **positive** and **negative** frequencies

$$c_n^{+}=-I_n^{(+-)}, \qquad \boldsymbol{c_n^{-}=I_n^{(++)}}.$$

which leads to **particle production.**

Employing the **WKB aproximation**, the corresponding particle production only depends on the background and it is exponentially amplified.

Conclusions

- A set of criteria to SELECT a preferred **UNIQUE CLASS** of Fock quantizations for scalar fields in a variety of nonstationary spacetimes with compact spatial topology
- Removing the ambiguities provides physical predictions with great robustness.
- **Generalization** to all the second order equations of motion, through the combination of a scaled field configuration and a time reparametrization, univocally determined.
- **Space-time interpretation** of the considered equation of motion, as fields propagating in conformally ultrastatic spacetimes.
- **Signature change** ⟶ elliptic rather than hyperbolic partial differential equations for physical modes.

  ⟶ space-time **singularity**: there exists a point where the metric is totally degenerated and the scalar invariant curvature becomes infinity.
- Evolution of a vacuum state from a Euclidean to a Lorentzian region.
- Generally, there exists an exponentially amplified **“particle production”**.


THANK YOU
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